arXiv:cond-mat/0404716v2 [cond-mat.mes-hall] 25 May 2004

How to increase a transmission with weak localization ? A geometrical effect

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We study the quantum transport in multiterminal networks of quasi-one-dimensional diffusive wires. When calculating the weak localization correction to the conductances, we show that the Cooperon must be properly weighted over each wire. This can even change the sign of the weak localization correction in certain geometries.

1 Introduction

The classical conductance of a wire of length L and section s made of weakly disordered metal is given by the Ohm's law $G_{\rm D} = \sigma_0 s/L$, where σ_0 is the Drude conductivity. Quantum interferences are responsible for a small *negative* contribution, in the absence of spin-orbit coupling, that shows up at sufficiently low temperature, the so-called weak localization (WL) correction. This effect, which is destroyed by a strong magnetic field, is particularly interesting from the experimental point of view because it provides an efficient way to extract a phase coherence length L_φ in a disordered metal, by studying its magnetoconductance. In the middle of the 80's, the progresses in nanolithography allowed to realize not only wires of mesoscopic sizes (μm) but also networks of wires, whose more complicate topologies make them particularly suitable to study interference effects. The first experiments were realized on large honeycomb metallic lattices [1] and showed the AAS oscillations predicted in [2]. Many other experiments have been realized until then on necklace of loops, ladders, square lattices,... The first theoretical description of WL in networks was provided by Douçot & Rammal (DR) [3]and was successfully used to describe the experiments of Pannetier *et al*[1]. The starting point of this approach is a uniform integration of the Cooperon $\Delta\sigma(\vec r) = -\frac{e^2}{\pi} P_c(\vec r, \vec r)$ over the system [4]. $P_c(r,r)$ is the return probability of a diffusion problem. However, it is meaningful to define a *local* conductivity ($\sigma = \frac{1}{\rm Vol}\int {\rm d}\vec r {\rm d}\vec r\,'\, \sigma(\vec r, \vec r\,'))$ only if the system is regular. If instead one considers a network of arbitrary topology, like on figure 1, its tranport properties are conveniently described, in the Landauer-Büttiker formalism, by transmission probabilities $T_{\alpha'\beta'}$ to go from a contact β' to a contact α', which are related to the *nonlocal* conductivity. The transmissions averaged over the disorder are written $\langle T_{\alpha'\beta'}\rangle = T^{\rm cl}_{\alpha'\beta'} + \Delta T_{\alpha'\beta'} + \cdots$ We have demonstrated in [6] that the Cooperon must be properly weighted when integrated over the wires $(\mu\nu)$ of the network. These weights depend on the topology and the way the network is connected to reservoirs. Moreover, we showed that the geometrical weight to attribuate to a wire $(\mu\nu)$ is given by the derivative of the classical transmission $T^{\rm cl}_{\alpha'\beta'}$ with respect to the length $l_{\mu\nu}$:

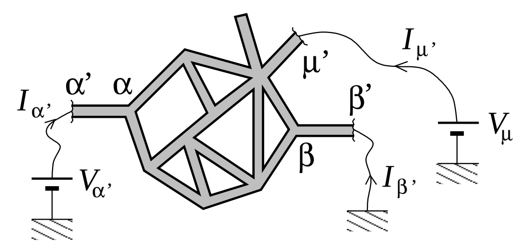


Figure 1: A network of diffusive wires. The network is connected at the vertices α', β' and μ' to external reservoirs (wavy lines) through which some current is injected in the network.

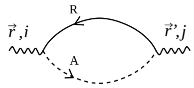
$$\Delta T_{\alpha'\beta'} = \frac{2}{\alpha_d N_c \ell_e} \sum_{(\mu\nu)} \frac{\partial T^{\rm cl}_{\alpha'\beta'}}{\partial\, l_{\mu\nu}} \int_{(\mu\nu)} {\rm d}x\, P_c(x,x)\,. \tag{1}$$

N_c is the number of conducting channels per wire, ℓ_e the elastic mean free path and α_d a numerical constant depending on the dimension ($\alpha_1 = 2$, $\alpha_2 = \pi/2$ and $\alpha_3 = 4/3$). For regular networks the uniform integration of P_c is justified, however the weights cannot be neglected in general : In multiterminal geometries they can even *change the sign of the WL correction.*

2 Transport in weakly disordered metals

The classical transport is described by two terms : the Drude contribution (short range) and the contribution from the diffuson (ladder diagrams) which is long range. In the diffusion approximation, we have [7] :

$$\langle \sigma_{ij}(\vec{r},\vec{r}\,')\rangle_{\rm class} =$$



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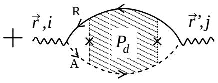
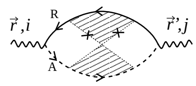


$$= \sigma_0 \left[\delta_{ij}\,\delta(\vec{r}-\vec{r}\,') - \nabla_i \nabla'_j P_d(\vec{r},\vec{r}\,')\right] \,, \tag{2}$$

where the diffuson P_d is solution of the equation $-\Delta P_d(\vec{r},\vec{r}\,') = \delta(\vec{r}-\vec{r}\,')$. We introduce the notation $\langle \sigma_{ij}(\vec{r},\vec{r}\,')\rangle_{\rm class} = \sigma_0\,\phi_{ij}(\vec{r},\vec{r}\,')$. An important requirement of a transport theory is to satisfy current conservation $\nabla_i \sigma_{ij}(\vec{r},\vec{r}\,') = 0$, what the classical conductivity (2) does.

The WL correction is given by the maximally crossed diagrams [8,9] :

$$\langle \sigma_{ij}(\vec{r},\vec{r}\,')\rangle_{\rm cooperon} =$$



$$= -\frac{e^2}{\pi}\,\delta_{ij}\,\delta(\vec{r}-\vec{r}\,')\,P_c(\vec{r},\vec{r})\,, \tag{3}$$

where the Cooperon is solution of $[\frac{1}{L_\varphi^2} - (\vec{\nabla} - 2{\rm i}e\vec{A})^2]P_c(\vec{r},\vec{r}\,') = \delta(\vec{r}-\vec{r}\,')$. It is clear that the additional contribution of the Cooperon (3) does not respect current conservation. In the same way that the classical conductivity is built of short range (Drude) and long range (diffuson) contributions, the WL correction contains long range terms additionally to the short range contribution (3). A convenient method to construct the current conserving quantity was proposed in [7], that avoids appearance of divergencies in the long range contribution. It leads to [6]

$$\langle \Delta\sigma_{ij}(\vec{r},\vec{r}\,')\rangle = \int {\rm d}\vec{\rho}\,{\rm d}\vec{\rho}\,'\;\phi_{ii'}(\vec{r},\vec{\rho})\,\phi_{jj'}(\vec{r}\,',\vec{\rho}\,')\,\langle \sigma_{i'j'}(\vec{\rho},\vec{\rho}\,')\rangle_{\rm cooperon}\,. \tag{4}$$

3 Networks

We now consider specifically the case of networks such as the one on figure 1. The transmission $T_{\alpha'\beta'}$ is related to the nonlocal conductivity with the two arguments at contacts α' and β' (see details in [6,10]). For quasi one-dimensional (1d) wires, we obtain the expressions

$$T^{\rm cl}_{\alpha'\beta'} = \frac{\alpha_d N_c}{\ell_e} P_d(\underline{\alpha}',\underline{\beta}') \quad \text{and} \quad \Delta T_{\alpha'\beta'} = \frac{2}{\ell_e^2}\int_{\rm Network} {\rm d}x\,\frac{\rm d}{{\rm d}x}P_d(\underline{\alpha}',x)\,P_c(x,x)\,\frac{\rm d}{{\rm d}x}P_d(x,\underline{\beta}')\,, \tag{5}$$

that involve the 1d diffuson and cooperon. The notation $P_d(\underline{\alpha}',x)$ means that P_d is taken at a distance ℓ_e of the vertex α' (the question of boundary conditions is discussed in detail in [6,10]). Since $P_d(\underline{\alpha}',x)$ is a linear function of x, the two diffusons in $\Delta T_{\alpha'\beta'}$ bring a constant that depends on the wire to which x belongs. In other terms, when integrated over a wire, P_c must be weighted by a coefficient depending on the wire, shown in [6] to be the one given in (1).

The next step is to construct explicitely $P_{d,c}$ in the network. To describe its topology we introduce the adjacency matrix $a_{\alpha\beta}$: we have $a_{\alpha\beta} = 1$ if the vertices α and β are connected by a wire and $a_{\alpha\beta} = 0$ otherwise. The parameters λ_α describe how the network is connected : $\lambda_\alpha = 0$ for an internal vertex and $\lambda_{\alpha'} = \infty$ at the vertices connected to external reservoirs (the connected vertices are primed). The magnetic flux along the wire $(\alpha\beta)$ is denoted $\theta_{\alpha\beta}$. The solution for P_c involves the matrix [3,5,11,10] :

$$\mathcal{M}_{\alpha\beta} = \delta_{\alpha\beta}\left(\lambda_\alpha + \sqrt{\gamma}\sum_\mu a_{\alpha\mu}\coth(\sqrt{\gamma}l_{\alpha\mu})\right) - a_{\alpha\beta}\frac{\sqrt{\gamma}\,{\rm e}^{-{\rm i}\theta_{\alpha\beta}}}{\sinh(\sqrt{\gamma}l_{\alpha\beta})}\,. \tag{6}$$

The diffuson is expressed in terms of the same matrix with $\gamma = 0$ and no magnetic flux $\theta_{\alpha\beta} = 0$:

$$(\mathcal{M}_0)_{\alpha\beta} = \delta_{\alpha\beta}\left(\lambda_\alpha + \sum_\mu a_{\alpha\mu}\frac{1}{l_{\alpha\mu}}\right) - a_{\alpha\beta}\frac{1}{l_{\alpha\beta}}\,. \tag{7}$$

This matrix encodes the information about the classical conductances $\alpha_d N_c \ell_e / l_{\mu\nu}$ of each wire $(\mu\nu)$. Note that $\lambda_{\alpha'} = \infty$ (at a reservoir) implies that $(\mathcal{M}^{-1})_{\mu\alpha'} = 0$, $\forall\ \mu$. The classical conductance is given by

$$T^{\rm cl}_{\alpha'\beta'} = \frac{\alpha_d N_c \ell_e}{l_{\alpha\alpha'} l_{\beta\beta'}} \left(\mathcal{M}_0^{-1}\right)_{\alpha\beta} \,. \tag{8}$$

This result is only valid for $\alpha' \neq \beta'$ (see [12,10]). It coincides with the one obtained for a network of classical resistances, as it should. The integral of the Cooperon in (1) is a nonlocal quantity that carries information on the whole structure of the network through the matrix $\mathcal{M}^{-1}$:

$$\begin{aligned}\int_{(\mu\nu)} \mathrm{d}x\, P_c(x,x) = \frac{1}{2\sqrt{\gamma}} \Bigg\{ \left[\left(\mathcal{M}^{-1}\right)_{\mu\mu} + \left(\mathcal{M}^{-1}\right)_{\nu\nu} \right] \left(\coth\sqrt{\gamma} l_{\mu\nu} - \frac{\sqrt{\gamma} l_{\mu\nu}}{\sinh^2 \sqrt{\gamma} l_{\mu\nu}} \right) \\ + \left[\left(\mathcal{M}^{-1}\right)_{\mu\nu} \mathrm{e}^{\mathrm{i}\theta_{\mu\nu}} + \left(\mathcal{M}^{-1}\right)_{\nu\mu} \mathrm{e}^{\mathrm{i}\theta_{\nu\mu}} + \frac{\sinh\sqrt{\gamma} l_{\mu\nu}}{\sqrt{\gamma}} \right] \frac{-1 + \sqrt{\gamma} l_{\mu\nu} \coth\sqrt{\gamma} l_{\mu\nu}}{\sinh\sqrt{\gamma} l_{\mu\nu}} \Bigg\} \,.\end{aligned} \tag{9}$$

Both the classical transmissions and their WL corrections can be computed by algebraic calculations only from the two matrices.

4 Importance of the weights on two examples of networks

We now study two examples of networks where the existence of the weights cannot be neglected.

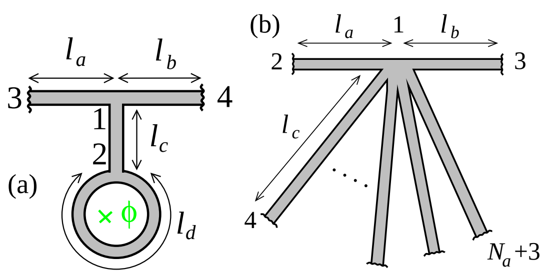


Figure 2 : Two examples of mesoscopic devices.

A ring.– The first example is the loop of figure 2a. This example emphasizes the nonlocality of the WL. The classical transmission $T^{\rm cl}_{34} = \frac{\alpha_d N_c \ell_e}{l_a + l_b}$ is independent of the presence of the arm and the ring. Then their weights are 0 and this part of the network does not contribute to the WL correction (1). However, since the cooperon $P_c(x,x)$ is *nonlocal*, it feels the presence of the loop, even for x in the wires $(3 \leftrightarrow 1)$ and $(1 \leftrightarrow 4)$. The harmonics of the AAS oscillations, $\Delta\hat{T}^{(n)}_{34} = \int_0^{\phi_0/2} \frac{\mathrm{d}\phi}{\phi_0/2} \Delta T_{34}(\phi) \mathrm{e}^{-4\mathrm{i}\pi n\phi/\phi_0}$, are found to be, in the limit $l_a, l_b \gg L_\varphi$:

$$\Delta\hat{T}^{(n)}_{34} \simeq -\left(\frac{L_\varphi}{l_a + l_b}\right)^2 \left(\frac{2}{3}\right)^{n+3} \mathrm{e}^{-2l_c/L_\varphi - n\, l_d/L_\varphi} \qquad \text{for } l_c, l_d \gg L_\varphi \tag{10}$$

$$\simeq -\left(\frac{L_\varphi}{l_a + l_b}\right)^2 \sqrt{\frac{l_d}{2L_\varphi}}\, \mathrm{e}^{-n\sqrt{2l_d/L_\varphi}} \qquad \text{for } l_c, l_d \ll L_\varphi \tag{11}$$

Note that the naive uniform integration of the cooperon over the network (DR&PM) strongly overestimates the amplitude of the AAS oscillations (figure 3) [10]. This is related to the fact that the uniform integration misses the $\mathrm{e}^{-2l_c/L_\varphi}$ factor brought by the Cooperon exploring the ring starting from the wire. The decrease of the WL at high field (inset) is due to the contribution of the flux to the effective phase coherence length [13] $L_\varphi(\phi)$. The behaviour $\mathrm{e}^{-n\, l_d/L_\varphi}$ in (10) comes from the normal diffusion in a ring (n_t, the typical number of turns after a time t behaves like $n_t \sim t^{1/2}$). On the other hand, for a very coherent ring ($l_d \ll L_\varphi$) connected to long wires ($l_a, l_b \gg L_\varphi$), the change of behaviour in (11) originates from a subdiffusive motion ($n_t \sim t^{1/4}$) due to the exploration of the long wires ($\gg l_d$) : each time the diffusive trajectory encircles the loop, the diffusion in the wires increases the effective perimeter, that now reads $l_{\rm eff} = l_d \sqrt{2L_\varphi / l_d}$. The same phenomenon occurs in the study of the transport through a ring. This illustrates that the connecting wires must be properly taken into account.

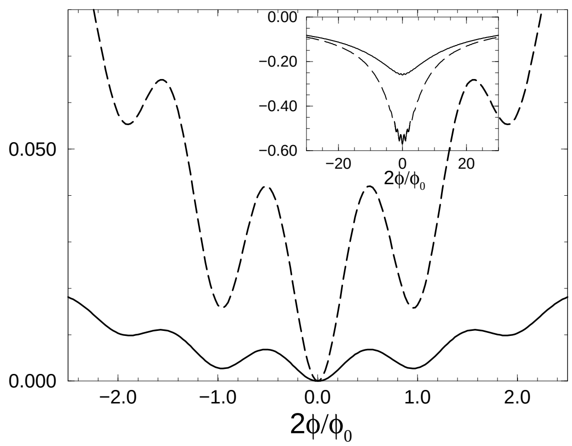


Figure 3 :
Magnetoconductance for the ring of figure 2a. Dashed line : $\Delta\sigma/\sigma_0$ for a uniform integration of P_c. Continuous line : $\Delta T_{34}/T_{34}^{\rm cl}$ given by (1). The curves have been shifted so that they coincide at $\phi=0$.
Inset : Same curves (without shift) for a higher window of flux ϕ. The parameters are $l_a=l_b=1\ \mu$m, $l_c=0.05\ \mu$m and $l_d=5\ \mu$m. $W=0.19\ \mu$m, $L_\varphi(\phi=0)=1.7\ \mu$m.

Multiterminal geometry.– The origin of the negative WL correction $\Delta T_{\alpha'\beta'}$ lies in the negative weights, however for a multiterminal geometry some weights can be *positive*, like the weight(s) $\partial T^{\rm cl}_{\alpha'\beta'}/\partial l_{\mu\mu'}$ of the wire(s) connected to other terminal(s) (*e.g.* μ' on figure 1). As a first example, we consider a wire on which is plugged one long arm of length l_c connected to a third reservoir (figure 2b for $N_a=1$). We focus ourselves on the fully coherent limit $L_\varphi=\infty$. The classical (Drude) conductance of this 3-terminal network is : $T^{\rm cl}_{23}=\frac{\alpha_d N_c \ell_e\, l_c}{l_a l_b+l_b l_c+l_c l_a}$. Then $\partial T^{\rm cl}_{23}/\partial l_c>0$. The wire $[(2\leftrightarrow1)+(1\leftrightarrow3)]$ gives a negative contribution to the WL correction whereas the arm $(1\leftrightarrow4)$ gives a *positive* one. Introducing $l^{-1}_{a//b//c}=l_a^{-1}+l_b^{-1}+l_c^{-1}$, we find

$$\Delta T_{23}=\frac{1}{3}\left(-1+\frac{l_{a//b//c}}{l_c}+\frac{l^2_{a//b//c}}{l_a l_b}\right)\underset{l_{a//b}\ll l_c}{\simeq}\frac{1}{3}\left(-1+\frac{l_{a//b}}{l_a+l_b}\right)\ . \tag{12}$$

We now consider the case of N_a long arms plugged in the middle of the wire ($l_a=l_b$) like on figure 2b, to maximize their effect [10]. We obtain :

$$\Delta T_{23}\simeq\frac{1}{3}\left(-1+\frac{N_a}{4}\right)\ , \tag{13}$$

a result valid for $l_a\ll l_c\ll L_\varphi$. We can now obtain a positive WL correction for $N_a>4$. This effect is purely geometrical. Note that in the limit $l_c\gg L_\varphi$ the positive contribution vanishes. The positive contribution of the arm in (12) can only be observed in a 3-terminal measurement, when the arm is a current sink. If the arm is a voltage probe, the positive contribution vanishes.

1. B. Pannetier, J. Chaussy, R. Rammal and P. Gandit, *Phys. Rev. Lett.* **53**, 718 (1984); *Phys. Rev.* B **31**, 3209 (1985).
2. B. L. Al'tshuler, A. G. Aronov, and B. Z. Spivak, *JETP Lett.* **33**, 94 (1981).
3. B. Douçot and R. Rammal, *Phys. Rev. Lett.* **55**, 1148 (1985); J. Physique **47**, 973 (1986).
4. This approach has been made more efficient by Pascaud & Montambaux (PM) in [5].
5. M. Pascaud and G. Montambaux, *Phys. Rev. Lett.* **82**, 4512 (1999).
6. C. Texier and G. Montambaux, *Phys. Rev. Lett.* **92**, 186801 (2004).
7. C. L. Kane, R. A. Serota and P. A. Lee, *Phys. Rev.* B **37**, 6701 (1988).
8. L. P. Gor'kov, A. I. Larkin and D. E. Khmel'nitzkiĭ, *JETP Lett.* **30**, 228 (1979).
9. B. L. Al'tshuler, D. E. Khmel'nitzkiĭ, A. I. Larkin, and P. A. Lee, *Phys. Rev.* B **22**, 5142 (1980).
10. C. Texier, G. Montambaux and E. Akkermans. *in preparation*, 2004.
11. E. Akkermans, A. Comtet, J. Desbois, G. Montambaux and C. Texier, Ann. Phys. (N.Y.) **284**, 10 (2000).
12. M. B. Hastings, A. D. Stone and H. U. Baranger, *Phys. Rev.* B **50**, 8230 (1994).
13. B. L. Al'tshuler and A. G. Aronov, *JETP Lett.* **33**, 499 (1981).